University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2018**

Problem Set #2

(due 11/27/18)

1. In the Harberger two-sector model, with overall supplies of labor and capital fixed and earning rates of return w and r, respectively, labor bears a fraction ψ of an incremental tax burden Δ if the ratio

$$R = \frac{wL + \psi\Delta}{wL + rK + \Delta}$$

is unchanged as Δ increases from its initial value of 0; that is, $dR/d\Delta = 0$.

   A. Show that labor's share of the burden, ψ, equals it share of initial income, $wL/(wL+rK)$, if there is no change in the ratio w/r (i.e., $\hat{w} - \hat{r} = 0$) as the tax is introduced.

   B. Suppose that the tax introduced is on capital income in sector X, so that $\Delta = T_{KX} r K_X$. Derive a condition for $\hat{w} - \hat{r} = 0$, using the expression for $\hat{w} - \hat{r}$ derived in Lecture Note 7 for this tax experiment.

   C. Now suppose that $\sigma_D = \sigma_X$ and that sector X uses both capital and labor in production. Show that the condition you derived in part B cannot be satisfied, and hence that $\hat{w} - \hat{r} > 0$: capital's relative share of the tax <u>must</u> be higher than labor's. (*Hint*: you will need to use the fact that $a_X = \lambda_{LX}\theta_{KX} + \lambda_{KX}\theta_{LX}$).

2. In class, we showed that a consumption tax is equivalent to a tax on labor income plus a tax on existing assets. This question reconsiders the issue in the case of nominal and real assets.

   A. Write down the budget constraint, expressing consumption in terms of labor income and assets, for a household that lives for two periods, supplies labor L in the first period for real wage w, has initial real assets in the first period, A, and consumes goods in both periods, c_1 and c_2, with price level p in both periods applicable to all real quantities (i.e., there is no inflation). Assume the household faces a tax at rate t on capital income and labor income and that saving yields a before-tax return r.

   B. Suppose now that the household initially holds two types of assets, real assets A and nominal bonds with nominal value B, each yielding r. Rewrite the budget constraint for this case, assuming again that the price level is p in both periods.

   C. Suppose that, at the beginning of period 1, the government replaces the income tax with a sales tax at rate τ on consumption in both periods, and that the real wage (in terms of the producer price of consumption) and the real before-tax return, r, are unaffected by the tax. Also assume that the producer price level (i.e., the price level *net* of sales tax) remains equal to p in both periods. Rewrite the budget constraint from part B for this tax system, showing that the consumption tax is equivalent to a tax on labor income plus all existing wealth.

D. Now, change the assumption about the price level in part C. Suppose that, when the sales tax is imposed, the Fed uses monetary policy to keep the consumer price level (which now includes the sales tax) at its original value, rather than the producer price level. How does your answer to part C change, assuming again that the real wage and interest rate are unaffected by the tax reform?

E. Now, go back to the price level assumption from part C, but assume that bond interest is exempt from the initial income tax (as is true for bonds issued by U.S. state and local governments), so that in equilibrium under the income tax nominal bonds yield a before-tax return of $r(1\text{-}t)$. Suppose also that bonds are consols, i.e., of infinite duration with constant nominal payments over time. How does your answer to part C change?

3. Consider an individual who wishes to invest initial wealth, W, to maximize the utility of terminal wealth one period hence. The investor's problem consists of two decisions:

   (1) how much of this wealth to place in bonds, which yield a certain return, $i > 0$, and how much to invest in stocks, which yield a stochastic return $r \in [0, R]$, $E(r) = \bar{r} > i$;

   (2) how to allocate these assets between a taxable account and a tax-sheltered account.

   Interest on bonds held in the taxable account (TA) is taxed at rate τ ($0 < \tau < 1$), while equity returns are taxed at rate $\lambda\tau$ ($0 < \lambda < 1$). Assets placed in the tax-sheltered account (TSA) are tax-exempt. An amount up to $V < W$ may be placed in the tax-sheltered account.

   A. Derive the optimal portfolio, in terms of the amounts of debt and equity held in each account, for an individual who is risk neutral; perform the same exercise for an individual who is infinitely risk averse.

   B. Show that, regardless of an individual's degree of risk aversion, it will never be optimal for the individual to hold equity in the TSA and bonds in the TA at the same time. (Hint: starting with such an initial allocation, show that adjustments in the composition of the two accounts would permit the investor to achieve higher aggregate after-tax earnings on debt for a given distribution of aggregate after-tax earnings on equity.)

4. Suppose that two countries each use source-based taxes to finance public goods. Each country has one variable factor of production, capital, and an identical production function, $F(K) = log(K)$ and $F(K^*) = log(K^*)$, where the "*" denotes the foreign country. Each country has a representative agent who owns capital, in amounts equal to A and A^*. Capital flows freely between the countries, so that capital stocks in the home and foreign countries, K and K^* (with $K + K^* = A + A^*$), adjust to the point where after-tax marginal products of capital in the two countries are equal: $r = (1\text{-}\tau)F'(K) = r^* = (1\text{-}\tau^*)F'(K^*)$. In addition to capital income taxes, each country also claims all pure domestic profits, so that total government revenue in the home country is $[F(K) - rK]$, with a similar expression for the foreign country. Each government's revenue is spent on public goods, G and G^*. Each agent's private consumption equals asset income ($C= rA$ in the home country, $C^* = r^*A^*$ abroad), and governments seek to maximize domestic utility, $U = log(C) + \beta G$ and $U^* = log(C^*) + \beta^*G^*$, with $\beta, \beta^* \geq 1$.

A. Solve for the equilibrium after-tax rate of return ($r = r^*$) and capital stocks (K and K^*) as a function of the tax rates τ and τ^* and the asset endowments A and A^*.

B. Assuming that the choices of tax rates τ and τ^* are determined in a Nash equilibrium, derive the first-order condition for the home country's choice of τ in terms of β and τ^*. (You do not have to solve explicitly for τ, just simplify the first-order condition.)

C. Using the first-order condition you derived in part B, show that the two taxes are strategic complements, i.e., that $d\tau/d\tau^* \geq 0$. (Assume that $\tau^* \geq 0$.)

D. Suppose that the two countries are identical in terms of preferences ($\beta^* = \beta$) and endowments ($A = A^*$). Solve for the optimal home tax rate, τ, assuming a symmetric equilibrium.